

## Comparative analysis of the Atkinson and the Otto cycles with heat transfer, friction and variable specific heats of working fluid

Mohammad Mehdi Rashidi<sup>1</sup>, Alireza Hajipour<sup>2\*</sup>, Afshin Fahimirad<sup>3</sup>

<sup>1</sup>Associate Professor at the Engineering Faculty of Bu-Ali Sina University, Department of Mechanical Engineering, Hamedan, Iran, *mm\_rashidi@yahoo.com*

<sup>2\*</sup>Corresponding author: MSc of Mechanical Engineering, Young Researchers Club, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran, *alirezahajipour@gmail.com*

<sup>3</sup>MSc of Mechanical Engineering, CHP System Project Manager, Iran Heavy Diesel Mfg. Co. (Desa), Amol, Iran, *a.fahimirad@desa.ir*

### Abstract

In this article, the performance of air standard Atkinson and Otto cycles with irreversibility factors, i.e. efficiencies of compression and expansion processes, heat transfer loss, friction-like term loss and variable specific heats of working fluid is analyzed using finite-time thermodynamics. The relations between the power output and the compression ratio and between the thermal efficiency and the compression ratio of the cycle are derived by detailed numerical examples. Moreover, the effects of heat transfer loss, the friction-like term loss and variable specific heats of working fluid on the cycle performance are analyzed. The results show that the effects of heat transfer loss, friction and variable specific heats of working fluid on the irreversible cycle performance should be considered in cycle analysis.

**Keywords:** Finite-time thermodynamics; Atkinson cycle; Otto cycle; Irreversibly; Efficiency

### 1- Introduction

There are some types of the four-stroke engines in which they are nearly associated to each other, with some differences in their design. The internal combustion engines which we focused on this paper are the Atkinson cycle and the Otto cycle. The Atkinson cycle engine was developed in 1882 by James Atkinson and the Otto cycle engine which was developed in 1876 by Nikolaus August Otto is an idealized thermodynamic cycle. Some authors have examined the finite-thermodynamic performance of the Atkinson and the Otto cycles. Comparison of performances of air standard Atkinson and Otto cycles with heat transfer

considerations is done by Hou [1]. Efficiency of an Atkinson engine at maximum power-density and performance of an Atkinson cycle with heat transfer, friction effects and variable specific-heats of the working fluid and thermodynamic simulation of performance of an Otto cycle with heat transfer and variable specific heats of working fluid and reciprocating heat-engine cycles is analyzed by Chen et al [2-5]. The universal power and efficiency characteristics for irreversible reciprocating heat engine cycles are studied by Qin et al [6]. When irreversibility factors considered to cycle's analyze, the analysis of cycle is closer than to the practical situation of cycle's performance and is a good benchmark for comparison and analyze for air-standard cycles progressive in internal combustion engines. In this paper we will study the effects of variable specific-heats of working fluid on performance of an Atkinson and an Otto cycles with heat transfer and friction considerations.

## 2- Cycle Model

The T-s diagrams of the air standard Atkinson and Otto cycles are shown in Figures 1-a and 1-b. The adiabatic compression and expansion processes are the same in the Atkinson and Otto cycles. The process (1 → 2s) and (3 → 4s) are the reversible adiabatic compression and expansion, while the process (1 → 2) and (3 → 4) are the irreversible adiabatic process in which consider into the thermal irreversibility in the real compression and expansion process. For the two adiabatic processes (1 → 2) and (3 → 4), the compression and expansion efficiencies can be expressed as

$$\eta_{compression} = \eta_c = \frac{(T_{2s} - T_1)}{(T_2 - T_1)} \quad (1)$$

$$\eta_{expansion} = \eta_e = \frac{(T_4 - T_3)}{(T_{4s} - T_3)}. \quad (2)$$

In the Atkinson cycle the heat added in the isochoric process (2 → 3) and the heat rejected in the isobaric process (4 → 1) and in the Otto cycle the heat addition and rejection processes occur in the isochoric states (2 → 3), (4 → 1). In Fig 1, we can assume that heating from state 2 to state 3 and cooling from state 4 to state 1 proceed at to constant temperature-rates, i.e.

$$\frac{dT}{dt} = \frac{1}{K_1} \text{ (for } 2 \rightarrow 3) \text{ and } \frac{dT}{dt} = \frac{1}{K_2} \text{ (for } 4 \rightarrow 1), \quad (3)$$

where  $T$  is the absolute temperature and  $t$  is the time;  $K_1$  and  $K_2$  are constants. Integrating Eq. (3) yields

$$t_1 = K_1(T_3 - T_2) \text{ and } t_2 = K_2(T_4 - T_1), \quad (4)$$

where  $t_1$  and  $t_2$  are the heating and cooling times, respectively. Then, the cycle period is

$$\tau = t_1 + t_2 = K_1(T_3 - T_2) + K_2(T_4 - T_1). \quad (5)$$

In practical cycles, specific heats of working fluid are variable and these variations will have great influences on the performance of the cycle. According to references [2-5], it can be supposed that the specific heats of working fluid are dependent on the temperature alone and, over the temperature ranges generally encountered for gases in heat engines (300-2200 K), the specific heat curve is nearly a straight line, which may be closely approximated by the following forms:

$$C_{pm} = a_p + k_1 T \quad (6)$$

$$C_{vm} = b_v + k_1 T, \quad (7)$$

where  $a_p$ ,  $b_v$  and  $k_1$  are constant,  $C_{pm}$  and  $C_{vm}$  are molar specific heats with constant pressure and volume, respectively. Accordingly, one has

$$R = C_{pm} - C_{vm} = a_p - b_v, \quad (8)$$

where  $R$  is the gas constant of the working fluid. For the Atkinson and the Otto cycles, the heat added to the working fluid during process (2 → 3) is

$$Q_{in,A,O} = M \int_{T_2}^{T_3} C_{vm} dT = M \int_{T_2}^{T_3} (b_v + k_1 T) dT = M \left[ b_v (T_3 - T_2) + 0.5 k_1 (T_3^2 - T_2^2) \right], \quad (9)$$

where  $M$  is the molar number of the working fluid. The heat rejected by the working fluid during process (4 → 1) for the Atkinson and the Otto cycles in a row are

$$Q_{out,A} = M \int_{T_1}^{T_4} C_{pm} dT = M \int_{T_1}^{T_4} (a_p + k_1 T) dT = M \left[ a_p (T_4 - T_1) + 0.5 k_1 (T_4^2 - T_1^2) \right], \quad (10)$$

and

$$Q_{out,O} = M \int_{T_1}^{T_4} C_{vm} dT = M \int_{T_1}^{T_4} (b_v + k_1 T) dT = M \left[ b_v (T_4 - T_1) + 0.5 k_1 (T_4^2 - T_1^2) \right]. \quad (11)$$

The work output of the cycle is

$$W = Q_{in} - Q_{out}. \quad (12)$$

Since  $C_{pm}$  and  $C_{vm}$  are dependent on temperature, the adiabatic exponent  $k = C_{pm} / C_{vm}$  will vary with temperature as well. Therefore, the equation often used in reversible adiabatic processes with constant  $k$  cannot be used in reversible adiabatic process with variable  $k$ . However, according to references [2-5], a suitable engineering approximation for a reversible adiabatic process with variable  $k$  can be made, i.e., this process can be broken up into

infinitesimally small process, and for each of these processes, the adiabatic exponent  $k$  can be regarded as a constant. For example, any reversible adiabatic process between states  $i$  and  $j$  can be regarded as consisting of numerous infinitesimally small processes with constant  $k$ . For any of these processes, when small changes in temperature  $dT$  and in volume  $dV$  of the working fluid take place, the equation for a reversible adiabatic process with variable  $k$  can be written as follows:

$$TV^{k-1} = (T + dT)(V + dV)^{k-1}. \quad (13)$$

From Eq. (13), one gets

$$k_1(T_j - T_i) + b_v \ln\left(\frac{T_j}{T_i}\right) = -R \ln\left(\frac{V_j}{V_i}\right). \quad (14)$$

The compression ratio is defined as

$$r_c = \frac{V_1}{V_2}. \quad (15)$$

Therefore, equations for process  $(1 \rightarrow 2)$  for the Atkinson and the Otto cycles as follows

$$k_1(T_{2s} - T_1) + b_v \ln\left(\frac{T_{2s}}{T_1}\right) = R \ln r_c. \quad (16)$$

For  $(3 \rightarrow 4)$  process for the Atkinson cycle

$$k_1(T_3 - T_{4s}) + b_v \ln\left(\frac{T_3}{T_{4s}}\right) + R \ln\left(\frac{T_1}{T_{4s}}\right) = R \ln r_c, \quad (17)$$

and for  $(3 \rightarrow 4)$  process for the Otto cycle

$$k_1(T_3 - T_{4s}) + b_v \ln\left(\frac{T_3}{T_{4s}}\right) = R \ln r_c. \quad (18)$$

For the ideals Atkinson and Otto cycles models there are no losses. However, for a real Atkinson cycle or Otto cycle, heat transfer irreversibilities between the working fluid and cylinder wall are not negligible. One can assume that the heat loss through the cylinder wall is proportional to average temperature of both the working fluid and cylinder wall, the wall temperature is assumed constant. The heat added to the working fluid by combustion process is given by the following relation [2-5]:

$$Q_{in} = M[\alpha - \beta(T_2 + T_3)], \quad (19)$$

where  $\alpha$  and  $\beta$  are two constant related to the combustion and heat transfer.

We can assume a dissipation term represented by a friction force, which as a function of the velocity, so

$$f_{\mu} = -\mu v = -\mu \frac{dx}{dt}, \quad (20)$$

where  $\mu$  is the coefficient of friction which takes into accounts the global losses and  $x$  is the piston displacement. Then, the lost power is

$$P_{\mu} = \frac{dW_{\mu}}{dt} = -\mu \frac{dx}{dt} \frac{dx}{dt} = -\mu v^2. \quad (21)$$

The piston's mean velocity is

$$\bar{v} = \frac{x_1 - x_2}{\Delta t_{12}} = \frac{x_2(r_c - 1)}{\Delta t_{12}}, \quad (22)$$

where  $x_2$  is piston's position at minimum volume and  $\Delta t_{12}$  is the time spent in the power stroke. Thus the lost power is:

$$P_{\mu} = b(r_c - 1)^2, \quad (23)$$

where

$$b = \frac{\mu x_2^2}{(\Delta t_{12})^2}. \quad (24)$$

So, the output power is

$$P = \frac{W}{\tau} - P_{\mu}. \quad (25)$$

The efficiency of the cycle is

$$\eta = \frac{P}{\frac{Q_{in}}{\tau}}. \quad (26)$$

When  $r_c$  and  $T_1$  are given,  $T_{2S}$  can be obtained from Eq. (16) for the Atkinson and the Otto cycles. We can obtain  $T_2$  from Eq. (1). Then, substituting Eq. (9) into Eq. (19) yields  $T_3$ , and  $T_{4S}$  can be worked out using Eq. (17) for the Atkinson cycle and Eq. (18) for the Otto cycle. Then  $T_4$  can be obtaining from Eq. (2) for both of cycles. Substituting  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  into Eqs. (25) and (26) yields the power and efficiency. Then, the relations between the power

output and the compression ratio, between thermal efficiency and the compression ratio can be derived.

### 3- Results and discussion

According to references [2-5], the following parameters are used in the calculations:

$\alpha = 60,000-70,000$  J/mol,  $\beta = 20-30$  J/mol.K,  $a_p = 28.182-32.182$  J/mol.K,  $b_v = 19.868-23.868$  J/mol.K,  $M = 1.57(10^{-5})$  kmol,  $T_1 = 300$  K,  $\eta_c = \eta_e = 0.985$  and  $k_I = 0.003844-0.009844$  J/mol.K<sup>2</sup>. The constant temperature rates  $K_1$  and  $K_2$  are estimated as  $K_1 = 8.128 (10^{-6})$  s/K and  $K_2 = 18.67 (10^{-6})$  s/K. In addition,  $b = 20-54$  W is set. Using the above constants and ranges of parameters, the characteristic curves of  $P$  versus  $r_c$ ,  $\eta$  versus  $r_c$  can be plotted.

Figs. 2 - 9 show the effects of the variable specific heats of the working fluid on the performance of the cycle with heat resistance and friction losses. One can see that the output power versus compression ratio characteristic and efficiency versus compression ratio characteristic are parabolic like curves. They reflect the performances characteristic of real irreversible Atkinson and Otto cycles.

From Eqs. (6) and (7), one can see that when  $k_I = 0$ ,  $C_{pm} = a_p$  and  $C_{vm} = b_v$  hold. It should be noted that  $a_p$  and  $b_v$  are constant specific heats of working fluid. Because  $R = C_{pm} - C_{vm} = a_p - b_v = \text{constant}$ ,  $a_p$  and  $b_v$  must change synchronously. Figs. 2 – 5 show the effects of  $a_p$  and  $b_v$  on the cycle performance. One can see that the maximum power output, the maximum efficiency will decrease with increase of  $a_p$  and  $b_v$ .

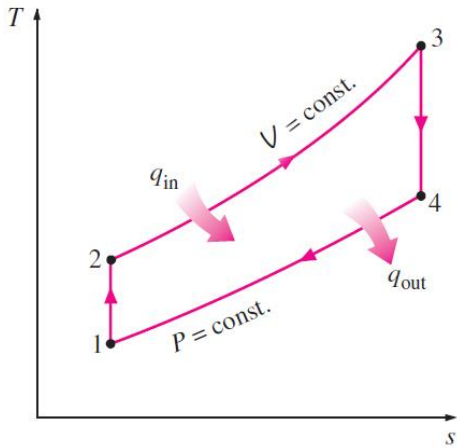
The parameter  $k_I$  reflects the variation degree of specific heat with temperature. The  $k_I$  is, the more acutely the specific heat varies with temperature. Figs 6 – 9 show the effects of  $k_I$  on the performances of cycles.

Figs. 10 – 13 show the effects of friction loss on the cycle performance. From Figs. 10 – 13, one can see that when  $b$  increases, the maximum power output, the maximum efficiency as well as the compression ratio will decrease. According to above analysis, the effects of the variable specific heats of working fluid on the Atkinson and Otto cycles performances are obvious, and they should be considered in practical cycle analysis in order to make the cycle model closer to practice.

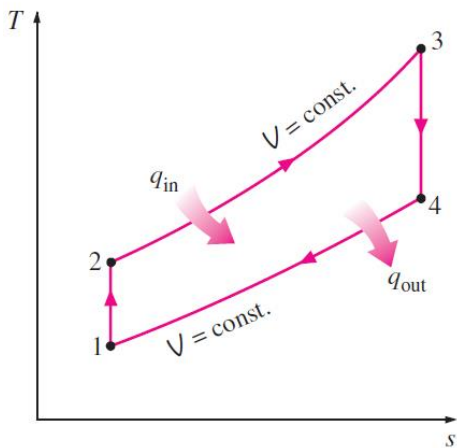
### 4- Conclusions

In this paper, the air standard Atkinson and Otto cycles, with considerations of heat transfer loss, friction like term loss and the variable specific heats of working fluid, is presented. The performances characteristic of these cycles were obtained by detailed numerical example. The results show that the effects of variable specific heats of working fluid on the cycle performance are significant, and should be considered in practical cycle analysis. The obtained results in this paper provide guidelines for the design of practical internal combustion engines.

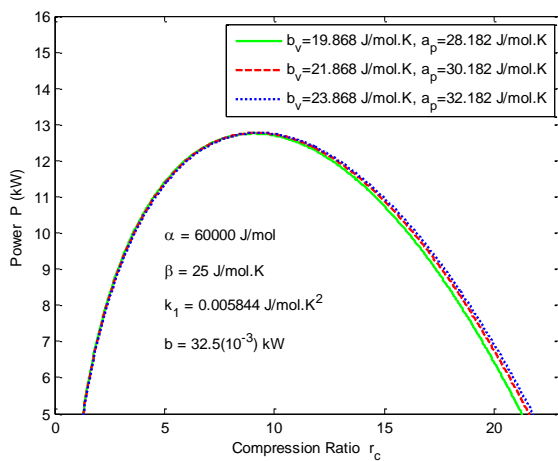
## 5- Figures



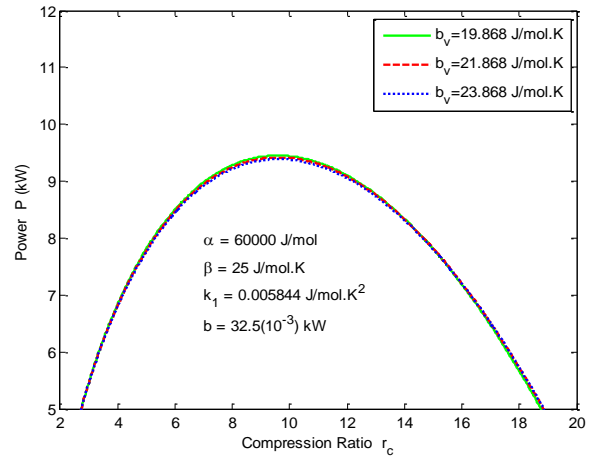
**Fig. 1-a** T-s diagram for an Atkinson cycle.



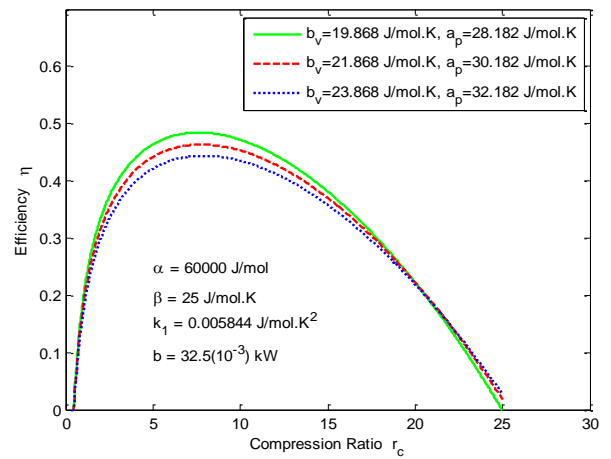
**Fig. 1-b** T-s diagram for an Otto cycle.



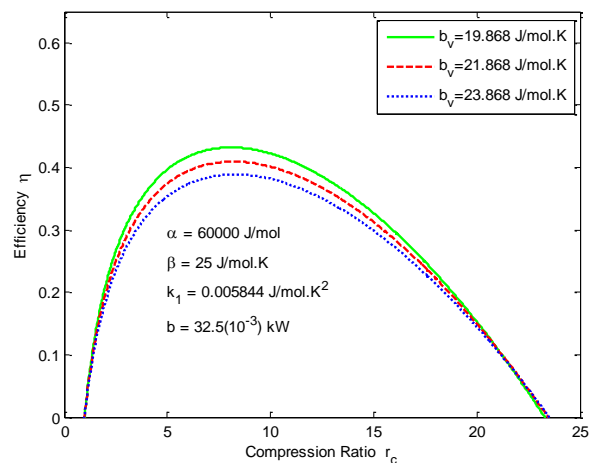
**Fig. 2** Influence of  $a_p$  and  $b_v$  on the output power of Atkinson.



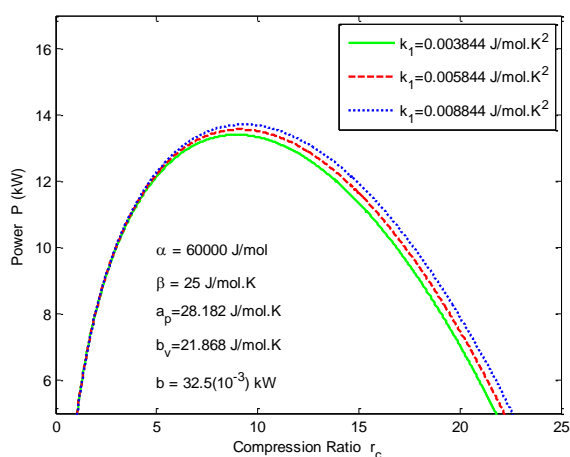
**Fig. 3** Influence of  $b_v$  on the output power of Otto.



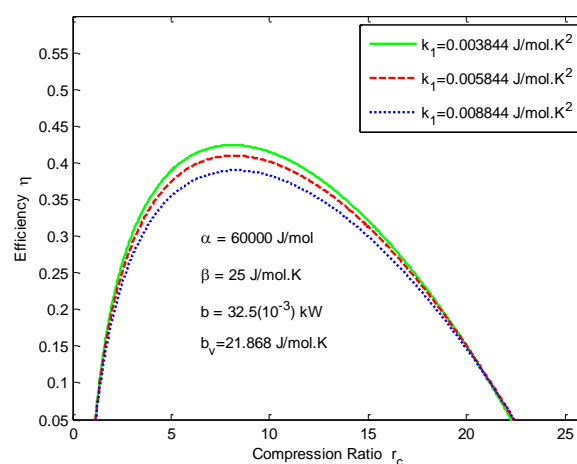
**Fig. 4** Influence of  $a_p$  and  $b_v$  on the efficiency of Atkinson.



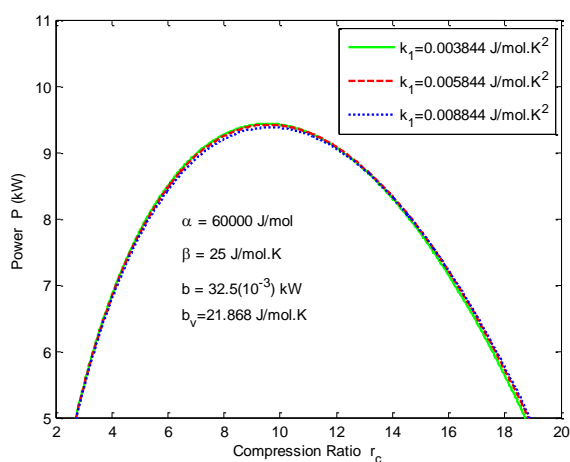
**Fig. 5** Influence of  $b_v$  on the efficiency of Otto.



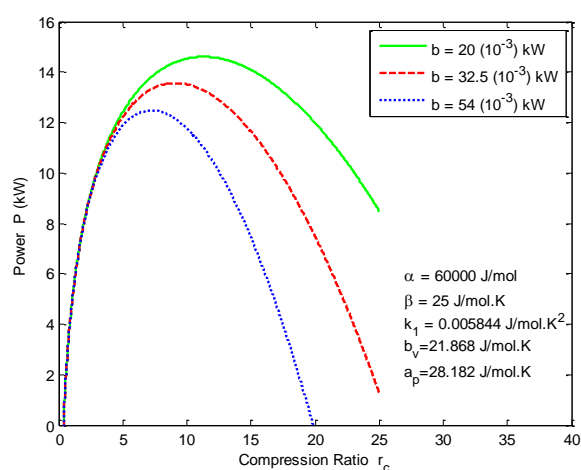
**Fig. 6** Influence of  $k_I$  on the output power of Atkinson.



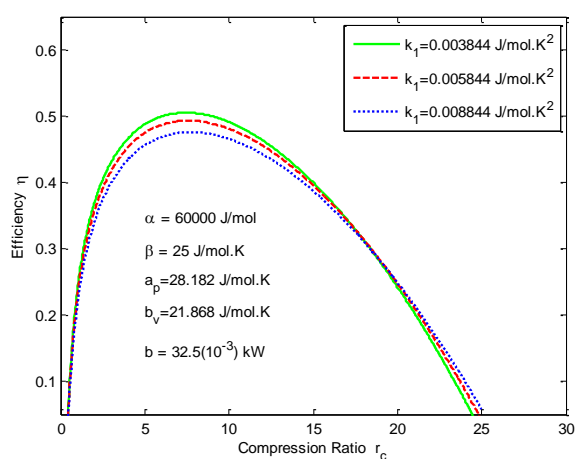
**Fig. 9** Influence of  $k_I$  on the efficiency of Otto.



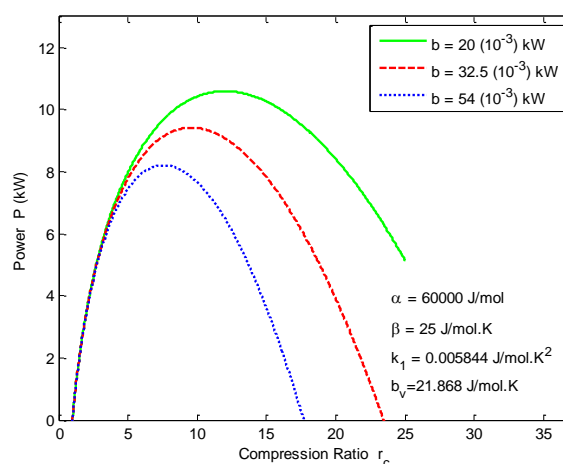
**Fig. 7** Influence of  $k_I$  on the output power of Otto.



**Fig. 10** Influence of  $b$  on the output power of Atkinson.

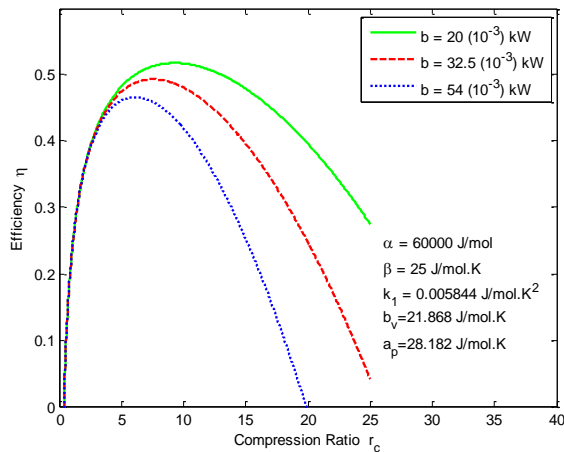


**Fig. 8** Influence of  $k_I$  on the efficiency of Atkinson.

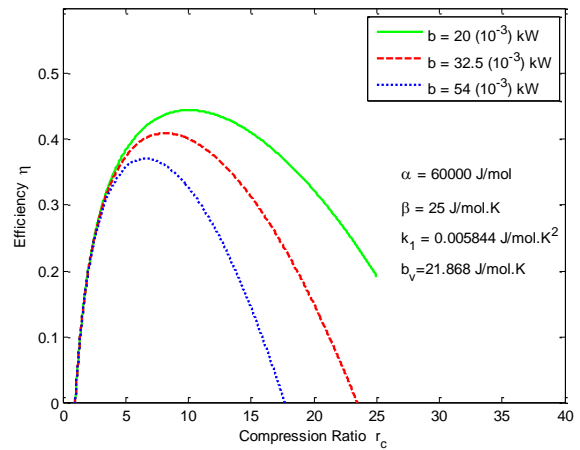


**Fig. 11** Influence of  $b$  on the output power of Otto.





**Fig. 12** Influence of  $b$  on the efficiency of Atkinson.



**Fig. 13** Influence of  $b$  on the efficiency of Otto.

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